A Constraint on the Biembedding of Latin Squares

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A biembedding of two latin squares of order n is equivalent to a face 2-colourable triangulation of $K_{n,n,n}$. We consider the following question: Given two latin squares A and B of order n, is there any relabelling of A and B for which there exists a biembedding?

Grannell, Griggs and Knor answered this question computationally for $n \leq 7$. A *main class* of latin squares is a set of latin squares which are equivalent under some relabelling. The number of main classes and distinct biembeddings increases rapidly with n, so that there is relatively little information for $n \leq 6$, while the problem is not computationally feasible for $n \geq 8$. For $n = 7$ a pattern emerged without parallel in the equivalent results for Steiner triple systems; the 147 main classes partition into 16 sets, such that a biembedding exists for most pairs of main classes from the same set, but there is no biembedding between any two latin squares which are not in the same set.

Using a argument based on permutation parity, we give a necessary condition explaining this pattern, and briefly explore some other implications of this result.

Based on joint work with Diane Donovan, Mike Grannell and Terry Griggs.